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ABSTRACT

Reported is an investigation to determine the nature and extent of student misconceptions in chemical equilibrium and to ascertain the degree to which certain misconceptions are related to chemistry achievement and to performance on specific tasks involving cognitive transformations characteristic of the concrete and formal operational stages of thought. The Misconception Identification Test (MIT), a 30-item multiple choice test, was developed to require the student to predict the effect of changing certain variables on the equilibrium conditions of selected chemical systems. Six major misconceptions were investigated: (1) mass vs. concentration, (2) rate vs. extent, (3) "constancy" of the equilibrium constant, (4) misuse of Le Chatelier's Principle, (5) constant concentration, and (6) competing equilibria. Ninety-nine grade-12 chemistry students in four classes (three teachers) participated in this study. Upon analysis of the data, the researchers concluded, among other findings, that students operating at the early or late concrete levels may benefit from a greater emphasis on a laboratory approach in which they can predict and then observe the effect of varying certain variables on a chemical system at equilibrium.
(Authors/PEB)

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STUDENT MISCONCEPTIONS IN CHEMICAL EQUILIBRIUM AS RELATED TO COGNITIVE LEVEL AND ACHIEVEMENT

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Introduction

In addition to requiring certain prerequisite concepts and skills in chemistry, high school treatments of chemical equilibrium e.g. CHEM Study tend to call for considerable abstraction and propositional thinking by the student. Recent studies in science teaching based on Piaget's theory of intellectual development (Buell and Bradley, 1972; Bass and Montague, 1972) suggest that many students do not always function at the cognitive level of which they are presumed capable. Some of the learning problems encountered by students in an area such as chemical equilibrium may be better understood in relation to their ability to deal with certain cognitive transformations associated with formal operational thought.

In order to describe how the adolescent manipulates data which he derives from experiments, Piaget has introduced a logical structure or model, the INRC group. (Inhelder and Piaget, 1958) The nature and extent of the use of the INRC transformations serves to distinguish the concrete operational child from the formal operational adolescent. Although the concrete operational child carries out operations on classes, relations, or numbers, his structure does not go beyond the level of elementary logical "groupings" or additive and multiplicative numerical groups. During the concrete stage, the child is capable of utilizing the two complementary forms of reversibility (inversion for classes and numbers and reciprocity for relations) but is unable to integrate them into the single total system. In contrast, the formal operational adolescent develops a mechanism which results in the integration of inversion and reciprocity.

The formal operational adolescent's thought structure is marked by a higher degree of reversibility than is present in previous stages. The two forms of reversibility, negation and reciprocity, become united in

a fully operational system. Another characteristic which distinguishes the formal operational stage from the concrete operational stage is the ability to identify all possible factors relevant to a problem under investigation by forming all possible combinations of these factors, one at a time, two at a time, three at a time, and so on. The individual need no longer confine his attention to what is real but can consider hypotheses that may or may not be true and work out what would follow if they were true. (Piaget, 1964) For purposes of the present study the INRC group model refers to the physical INRC group as clarified by Parsons (1960), and described by Flavell (1963, p. 217).

An example of formal operational thought is that required in coping with a problem in which the subject is given five bottles of colorless liquids, of which the first, third and fifth combine to form a yellow color, the second is neutral, and the fourth bleaches out the color (Inhelder and Piaget, 1958, p. 107-122). The problem is to find out how to produce the yellow color given the required solutions, labelled 1,2,3, 4, and "a", respectively. At the early concrete stage (C_1), subjects begin by mixing each solution with "a" or by taking them all at once. Although combinations are involved these are the most elementary and limited combinations that operate in multiplicative "groupings" of classes and relations. The idea of constructing combinations two-by-two or three-by-three, etc., does not occur at this level. In the later substage, C_2, the appearance of n-by-n combinations is noted. However, the subject does not as yet discover a system and only tentative empirical efforts are involved. The characteristic that these combinations are not systematic defines the upper limit of this substage, and subjects typically do not

investigate even the six possible two-by-two (with "c") combinations. The cause of the yellow color for the concrete operational subject is still sought in particular elements rather than in their combination. Although some subjects do locate the color by chance, the roles of other solutions are misinterpreted. The negative effect of solution 4 is also sometimes noted but a specific method of proof is lacking.

The two innovations which appear at the formal operational level are the systematic method in the use of n-by-n combinations and an understanding of the fact that the color is due to the combination as such. Formal stage subjects form their judgements according to a combinatorial system having the form of the sixteen binary propositions (Inhelder and Piaget, 1958). Combinations one-by-one, two-by-two, three-by-three, four or zero of the four base possibilities are taken. This formal mode of reasoning, founded on the combinations of factors, leads the subject to a new conception of the cause of the color. This cause is no longer sought in one or another of the solutions but in their being brought together, in the very fact of their combination. In the combination of chemical bodies problem, the difference between subjects as the two substanges of the formal level, designated F, and F_2, respectively, is one of degree, with the combinations emerging more rapidly as well as in a more systematic fashion in substange F_2.

Purpose

The purpose of this study was to determine the nature and extent of student misconceptions in chemical equilibrium and ascertain the degree to which certain misconceptions are related to chemistry achievement and to performance on specific tasks involving cognitive transformations characteristic of the concrete and formal operational stages of thought.

The Assessment of Misconceptualization

Paper-and-pencil approaches to the assessment of misconceptions have involved the development of item distractors according to predetermined misconception categories (Doran, 1972). Guttman and Schlesinger (1967) suggest the construction of distractors so as to vary in the degree of departure from the accepted answer along a given dimension. The present study used an item format in which the four response options were identical for all items.

The Misconception Identification Test (MIT) developed for this study is a 30-item multiple choice test which requires the student to predict the effect of changing certain variables eg. temperature, pressure, concentration, on the equilibrium conditions of selected chemical systems involving homogeneous gas reactions, phase changes, and aqueous solutions of ionic solids.* The questions are answered by choosing the most appropriate of the following responses a) greater than at the first equilibrium b) less than at the first equilibrium c) the same as at the first equilibrium d) there is insufficient evidence provided to decide among the above alternatives. The six major misconceptions under investigation were the following:

1. mass vs. concentration - inability to distinguish between the concepts of mass and concentration.
2. rate vs. extent - inability to distinguish between how fast a reaction proceeds (rate), and how far (extent) the reaction goes. (Driscoll, 1960)
3. "constancy" of the equilibrium constant - uncertainty as to when the equilibrium constant is in fact a constant.

*The authors wish to thank D.R. Driscoll, Canberra College of Advanced Education, Canberra City A.C.T. 2601 for suggestions regarding item format.

4. misuse of Le Chatelier's Principle - the application of 'Le Chatelier type' reasoning in inappropriate situations (Driscoll, 1960).

5. constant concentration - inability to appreciate that certain substances display a fixed or constant concentration in certain chemical reactions.

6. competing equilibria - inability to consider all possible factors affecting the equilibrium condition of a chemical system.

The MIT consists of six subtests corresponding to each of the six major misconceptions defined above and yields two scores. The performance score refers to the score a student obtains on a subtest of the MIT when it is keyed accurately in a chemical sense. The misconception score refers to the score a student obtains on a subtest of the MIT when it is keyed according to a given misconception. It should be noted that the performance score and the misconception score are closely related in that both scores are obtained from the same test. Once the performance score for a given student is obtained, restrictions are placed on the possible misconception score the student can obtain. Both the performance score and the misconception score for each student on each subtest of the MIT are used to determine whether or not the student possesses a particular misconception.

The interpretation of multiple choice type diagnostic items presents certain problems. A student can arrive at the correct answer either by guessing or by making a "wrong" mistake (so to speak) or by otherwise arguing incorrectly. He may also arrive at a particular incorrect answer by a variety of incorrect pathways.

If a student answers a given question incorrectly, this may not in itself reveal much. However, if he answers several questions which all deal with the same concept incorrectly, one could be reasonably sure that the student was having trouble with the particular point. Patterns of

responses for items dealing with the same misconception were examined and a misconception was said to be present if the misconception score accounted for 50 per cent or more of the subtest items answered incorrectly in the chemical sense. As an aid to interpretation, students were also asked to give free-response accounts of their reasoning for their predictions for five randomly chosen items on the MIT.

Many of the items on the MIT, in addition to their use in identifying major misconceptions in this area, involve particular transformations of the INRC group. The attempt to relate individual items to specific transformations revealed that this is a complex matter which probably cannot be accomplished by a priori logical analysis of the item alone. Although a preliminary categorization was carried out, detailed empirical data on student reasoning appears necessary.

Procedure

Ninety-nine grade twelve chemistry students in four classes were administered the following test instruments:

1. The Misconception Identification Test (MIT) developed for this study.
2. Chemistry Achievement Test (CHAT) based on Chapters 7, 8, 9, and 10 of the CHEM Study text. This is a 33 item five option multiple choice open book examination of 60 minute duration with a KR-20 of 0.75.
3. Two combinatorial tasks (PT1) involving the investigation of the nature of five colorless chemical solutions by manipulation of the various combinations of the solutions. The first task was based on the experiment described by Inhelder and Piaget (1958, p. 107-122). The second was devised by the experimenters and again involved the manipulation of five colorless solutions and the reproduction of a yellow coloration. The task was similar to the first in that a color-inhibiting solution was involved but the yellow color produced by the mixing of only two solutions. Also involved was an interfering starch solution which produced a deep blue color. The tasks were administered in a group format, a departure from the Piagetian interview technique which was considered defensible in view of the relationship between student performance on a written Piagetian test and an oral one (Gray, 1973). Performance

was evaluated on the basis of criteria outlined by Inhelder and Piaget (1958).

4. A written test (SK6) consisting of three 15-item sections based on ten operations involving the four transformations of the INRC physical group model (Skemp, 1961).

Testing commenced shortly after the three teachers involved completed the relevant CHEM Study chapters. The CHAT was administered first, followed by the MIT during the regular class period the next day. Both tests were used by the three teachers involved as part of their regular evaluation of chemistry achievement. The combinatorial tasks and the Skemp test were administered approximately one week later.

Results

The means, standard deviations, and intercorrelations of the tests are presented in Table I.

TABLE I

Intercorrelations, Means and Standard Deviations of the Test Scores

	MIT-A	MIT-B	CHAT	SK6-1	SK6-2	SK6-3	SK6-T	PT1
MIT-A*	1.00							
MIT-B**	.71							
CHAT	.55	.51						
SK6-1	.15	.17	.24					
SK6-2	.32	.33	.47	.39				
SK6-3	.21	.08	.21	.45	.33			
SK6-T	.29	.25	.41	.79	.75	.76		
PT1	.61	.46	.74	.18	.38	.23	.34	
Mean	13.6	18.6	19.4	10.9	10.5	6.0	27.5	2.8
Std. Dev.	3.3	3.9	5.0	3.2	3.3	3.1	7.4	0.7

$r < 0.20$, n.s. at $p = 0.05$ level.

*MIT-A = performance score

**MIT-B = misconception score

The MIT-A yielded a KR-20 reliability of 0. 49, an outcome which was not entirely unexpected as the test measures several different attributes dealing with various types of equilibria and is on the difficult side. The KR-20 coefficient increased to 0.57 for MIT-B as a result of keying the items according to the misconceptions, suggesting that the MIT perhaps has some internal consistency for the purpose of misconception identification in this area of chemistry.

Table II presents the item data on the MIT with the items categorized according to misconception. The chance values were computed on the basis of equal probability of occurrence of each of the three chemically incorrect response alternatives.

(Table II about here)

The probability of students choosing the keyed misconception response was found to consistently exceed that expected by chance alone. (Chi square values for 25 of the item response distributions are significant at the p 0.01 level). Table III presents the distribution according to number of misconceptions held. Eighty-two per cent of the sample were found to possess three or more of the misconceptions identified.

TABLE III

Distribution of Sample Showing Number of Misconceptions Held

N = 99

	Number of Misconceptions Held					
	1	2	3	4	5	6
Number of Students	2	16	36	25	18	2

TABLE II

Item Response Distributions on the MIT:
Chance Values and Chi Squares for Incorrect Responses

(N=99)

Relevant Misconception (% having Misc.)	Item	Response A	B	C	D	Chance Value	χ^2	Prob.
Misc. 1								
Mass vs. conc.	4	*.70	.07	.23°	.00	(.10)	27.8	x
29%	5	*.83	.03	.11°	.03	(.06)	7.5	xx
	7	*.27	.49°	.15	.07	(.24)	42.0	x
	24	.06	*.80	.05°	.08	(.07)	0.7	xx
Misc. 2	1	*.43	.42°	.14	.00	(.19)	49.0	x
rate vs. extent	2	.21°	*.55	.18	.06	(.15)	8.4	x
29%	6	.08	*.68	.23°	.00	(.11)	26.4	x
	20	.12°	*.83	.05	.00	(.06)	14.8	x
	25	*.73	.15°	.06	.05	(.09)	7.0	xx
Misc. 3	3	*.31	.35°	.27	.05	(.23)	21.6	x
Constancy of K_{eq}	9	.16	.28°	*.51	.04	(.16)	18.0	x
47%	14	.10°	.08	*.80	.02	(.07)	5.2	xx
	19	.14	.34°	*.39	.12	(.20)	12.8	x
	29	.17	.27°	*.49	.04	(.17)	16.6	x
Misc. 4	10	.52°	.16	.14	*.16	(.28)	33.5)	x
Misuse of	11	.40°	.07	*.48	.04	(.17)	47.0	x
LeChatelier's	15	.75°	.20	.04	*.01	(.33)	84.1	x
Principle 95%	16	.22	.70°	.05	*.02	(.33)	70.3	x
	21	.88°	.04	.07	*.01	(.33)	137.6	x
Misc. 5	23	*.26	.10	.62°	.02	(.25)	86.0	x
Constant Con-	26	.07	.45°	*.42	.03	(.19)	58.6	x
centration	27	.06°	*.27	.63°	.03	(.24)	95.3	x
84%	30	.60	.07	*.28	.03	(.24)	86.8	x
Misc. 6	8	*.76	.06	.16°	.01	(.08)	15.2	x
Competing	12	.14	*.03	.81°	.02	(.32)	112.1	x
Equilibria	13	.11	*.27	.60°	.02	(.24)	80.1	x
60%	17	*.62	.19°	.17	.02	(.13)	13.6	x
	18	.20°	*.70	.06	.04	(.10)	15.2	x
	22	.21	.42	.23°	.12	(.19)	3.7	xx
	28	*.54	.26°	.17	.02	(.15)	19.6	x

*keyed answer

°keyed misconception
( ) chance value of incorrect responses
x = $p < .01$
xx = $p > .05$

The two combinatorial tasks were found to be equivalent when evaluated separately according to the specified criteria. In terms of cognitive functioning, the sample was classified as follows: early concrete, 3 students; late concrete, 24 students; early formal, 61 students; and late formal, 11 students. These results are in general agreement with other investigations in this area (Buell and Bradley, 1972; Stephens, McLaughlin and Mahaney, 1971.) Intercorrelations among the two forms of the MIT and the combinatorial tasks were significant at the $p < 0.01$ level (see Table I). Although total performance on the Skemp test was found to be significantly related to both forms of the MIT, the three sections vary considerably in their relationship to the MIT.

Tables IV and V present the results of stepwise regression analyses (Draper and Smith, 1966) for the prediction of MIT-A and MIT-B scores.

It appears that a considerable amount of the variance contributed by the PTl to prediction of MIT-B scores is common to the CHAT. Although PTl and CHAT are highly correlated ($r = 0.74$) the CHAT scores do not enter the regression equation for the prediction of MIT-A scores, supporting the notion that the two forms of the MIT do function to some extent as two distinct tests.

Table VI presents the results of a Chi square test of independence between the number of misconceptions held and cognitive level. The categorization of the misconceptions into three or less and more than three misconceptions was chosen as it divided the sample into two nearly equal groups.

TABLE IV

Prediction of MIT(A) Scores from a Combination of Piagetian Tasks, PT1, and SK6, and CHAT Scores

Prediction Variable Entering	F Value for Variable Entering	Total F Value	Probability Level for Last Variable Entering	R^2 (per cent)
PT1	58.2	58.2	< .001	37.3

The regression equation in raw-score form is given by:

$$\hat{Y}_{MIT\text{-}A} = 3.0 X_{PT1} + 5.0$$

TABLE V

Prediction of MIT(B) Scores from a Combination of Piagetian Tasks, PT1, and SK6, and CHAT Scores

Prediction Variable Entering	F Value for Variable Entering	Total F Value	Probability Level for Last Variable Entering	R^2 (per cent)
CHAT	34.7	34.7	< .001	26.4

The regression equation in raw-score form is given by:

$$\hat{Y}_{MIT\text{-}B} = 0.4 X_{CHAT} + 11.0$$

TABLE VI

Chi Square Test of Independence: Number of Misconceptions vs. Cognitive Level

Number of Misconceptions	Concrete	Formal	Total
3 or less	8	46	54
over 3	19	26	45
Total	27	72	99

Cognitive Level

$\chi^2 = 9.3$ (df=1)

$(.001 < p < .01)$

Consideration of the relationship between specific misconceptions and performance on the Piagetian tasks revealed that misconceptions 1 (mass vs. concentration) and 2 (rate vs. extent) were significantly related ($p < 0.05$) to cognitive level. These were the two least prevalent misconceptions, occurring in 29 per cent of the sample.

The results of a stepwise regression analysis predicting chemistry achievement as measured by CHAT from the Piagetian tasks are presented in Table VII.

TABLE VII

Prediction of CHAT Scores from a Combination of Piagetian Tasks: PT1, SK6(1), SK6(2), and SK6(3)

Predictor Variable Entering	F Value for Variable Entering	Total F Value	Probability Level for Last Variable Entering	R^2 (per cent)
PT1	115.3	115.3	.001	54.3
SK6(2)	10.3	68.3	.002	58.7

The regression equation in raw-score is given by:

$$\hat{Y}_{CHAT} = 0.35 X_{SK6(2)} + 4.9 X_{PT1} + 1.9$$

Both perceptions of aspects of the INRC group model and performance on the combinatorial tasks were found to function as significant predictors of chemistry achievement for the sample in the study, together predicting 58.7 per cent of the variance of the achievement scores.

The subjects were categorized into high, middle, and low achievement groups on the basis of performance on CHAT. Cutting points at 17 and 21 produced three nearly equal groups, with no significant difference on CHAT performance for males (64 per cent of the sample) and females (36 per cent of the sample).

Table VIII presents the distribution of the number of misconceptions within the three achievement groups.

TABLE VIII

Chi Square Test of Independence:
Number of Misconceptions vs. Achievement Group

Number of Misconceptions	High	Middle	Low	Total
Less than 3	(6.5) 13	(5.5) 3	(6.0) 2	18
3 or more	(29.2) 23	(24.8) 27	(27.0) 31	81
Total	36	30	33	99

Achievement

$\chi^2 = 12.37$, df = 2, $.001 < p < .01$

Fify-seven per cent of the students in the low achievement group possessed more than three misconceptions compared to 30 per cent in the high achievement group. Misconceptions 3 ("constancy" of the equilibrium constant), 4 (misuse of Le Chatelier's Principle) and 6 (competing equilibria) were found to be significantly related ($p < 0.05$) to achievement group. Misconception 6

was significant at the $p < 0.001$ level, suggesting that the inability to control all possible variables in problems of chemical equilibrium may affect achievement in chemistry. Although misconceptions 1 and 2 were found to be related to cognitive level, no relationship between these misconceptions and achievement was observed.

Discussion

The relationship between the nature and and extent of misconceptualization in chemical equilibrium, cognitive level, and achievement level presents a number of considerations for instruction in this area. The findings of the present study would suggest that before introducing the principle of chemical equilibrium formally, some assessment of each student's cognitive level be attempted. The measures used to determine the cognitive level of the subjects in this study (PT 1 and SK6) are appropriate to the context of a chemistry class and might be given beforehand with relatively little difficulty. For example, PT1 could serve as a relevant introductory laboratory exercise. This would both allow a general assessment of each student's cognitive level and provide an opportunity to discuss the basic chemical reactions involved in the various combinations of the solutions in that task. Students operating at the early or late concrete levels may benefit from a greater emphasis on a laboratory approach involving concrete situations in which they can predict and then observe the effect of varying certain variables on a chemical system at equilibrium. The concepts of mass, concentration, rate of reaction and extent of reaction seem to require consolidation for these students as well. Exercises and programmed materials dealing with these concepts at both the concrete and formal levels could be prepared.

The limitations contained in Le Chatelier's Principle might be more clearly illustrated by the use of numerous well chosen examples. Problems to which Le Chatelier type reasoning is not amenable should also be presented and discussed with the students. The relatively few problems on chemical equilibrium usually encountered in high school chemistry are typically qualitative in nature and easily resolved by Le Chatelier's Principle. The high prevalence of the misuse of Le Chatelier's Principle in the present study can be easily understood if students were in the practice of applying the Principle only to examples where it was previously known to give a correct answer. Perhaps a larger number of examples of equilibria, both qualitative and quantitative, should be made available to the students.

Graphical representations used in conjunction with the teaching of the concepts of constant concentration and the 'equilibrium constant' may be useful to the chemistry teacher in overcoming misunderstandings so often associated with these concepts. For example, concentration vs. time graphs could be introduced to illustrate the effect of introducing more reactant or product into a system at equilibrium. Plotting the change in concentration over time may help some students to more concretely visualize what is actually thought to happen in the process.

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